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Generation of even and odd nonlinear coherent states

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Abstract. We show that a class of even and odd nonlinear coherent states, defined as the eigenstates of the product of a nonlinear function of the number operator and the square of the boson annihilation operator, can be generated in the centre-of-mass motion of a trapped and bichromatically laser-driven ion. The nonclasscial properties of the states are studied.

1. Introduction

Coherent states are important in many fields of physics [1, 2]. Coherent states $|\alpha\rangle$, defined as the eigenstates of the harmonic oscillator annihilation operator $\hat{a}$, $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$ [3], have statistical properties like the classical radiation field. In a harmonic oscillator potential the centre of the coherent state wavepacket follows the classical trajectory. There exist states of the electromagnetic field whose properties, such as squeezing, higher-order squeezing, antibunching and sub-Poissonian statistics [4, 5], are strictly quantum mechanical in nature. These states are called nonclassical states. A special feature of the coherent states is that they are the only pure states which are classical. All the other pure states of the electromagnetic field are nonclassical.

A generalization of the coherent states was performed by q-deforming the basic commutation relation $[\hat{a}, \hat{a}^\dagger] = 1$ [6, 7]. A further generalization is to define states that are eigenstates of the operator $f(\hat{n})\hat{a}$,

$$f(\hat{n})\hat{a}|f, \alpha\rangle = \alpha|f, \alpha\rangle \tag{1}$$

where $f(\hat{n})$ is an operator-valued function of the number operator $\hat{n} = \hat{a}^\dagger\hat{a}$. These eigenstates are called nonlinear coherent states (NCS). In the linear limit, $f(\hat{n}) = 1$, the NCS become the usual coherent states $|\alpha\rangle$. The NCS were introduced, as f-coherent states, in connection with the study of an oscillator whose frequency depends on its energy [8, 9]. A class of NCS can be realized physically as the stationary states of the centre-of-mass motion of a trapped ion [10] and exhibit nonclassical features such as squeezing and self-splitting.

Superposition of coherent states gives rise to states with nonclassical properties. An important case is the superposition of the coherent states $|\alpha\rangle$ and $|-\alpha\rangle$, where the resultant states are eigenstates of the operator $\hat{a}^2$ [11]. The symmetric combination is the even coherent state (ECS), $|\alpha, +\rangle$, and its number state expansion is

$$|\alpha, +\rangle = [\cosh|\alpha|^2]^{-1/2} \sum_{n=0}^{\infty} \frac{\alpha^{2n}}{\sqrt{(2n)!}} |2n\rangle. \tag{2}$$

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The antisymmetric combination is the odd coherent state (OCS), $|\alpha, -\rangle$, given by

$$|\alpha, -\rangle = [\sinh|\alpha|^2]^{-1/2} \sum_{n=0}^{\infty} \frac{\alpha^{2n+1}}{\sqrt{(2n+1)!}} |2n+1\rangle. \tag{3}$$

The ECS has a squeezing effect but no antibunching effect. The OCS has an antibunching effect but no squeezing effect [12, 13]. Both the ECS and OCS have oscillatory photon number distribution. These states can be generated by various schemes: propagation in Kerr medium [14, 15], micromaser cavity experiments [16], quantum nondemolition experiments [17] and the motion of a trapped ion [18]. The ECS and OCS can be interpreted as Schrödinger cat states for appropriately large values of α [14].

The notion of the ECS and OCS has been generalized to the case of NCS [19, 20]. The even and odd nonlinear coherent states (ENCS and ONCS, respectively) are defined as the eigenstates of the operator $F(\hat{n})\hat{a}^2$, where $F(\hat{n})$ is an operator-valued function of the number operator $\hat{n}$. We denote the eigenstates as $|\alpha, F\rangle$, and they satisfy

$$F(\hat{n})\hat{a}^2|\alpha, F\rangle = \alpha|\alpha, F\rangle \tag{4}$$

where α is complex. The above equation gives rise to the recurrence relation

$$\langle n+2|\alpha, F\rangle = \alpha \frac{\langle n|\alpha, F\rangle}{F(n)\sqrt{(n+1)(n+2)}} \tag{5}$$

for $n = 0, 1, 2, 3 \ldots$, where the function $F(n)$ is obtained by replacing the number operator $\hat{n}$ in $F(\hat{n})$ by the integer n. The complex numbers $\langle n+2|\alpha, F\rangle$ $(n = 0, 1, 2, \ldots)$ are the expansion coefficients of the state $|\alpha, F\rangle$ in the harmonic oscillator basis. The above recurrence relation between the expansion coefficients yields

$$\langle 2n|\alpha, F\rangle = \alpha^n \frac{\langle 0|\alpha, F\rangle}{F(2(n-1))!!\sqrt{(2n)!}} \tag{6}$$

$$\langle 2n+1|\alpha, F\rangle = \alpha^n \frac{\langle 1|\alpha, F\rangle}{F(2n-1)!!\sqrt{(2n+1)!}} \tag{7}$$

where $F(2(n-1))!! = F(0)F(2)F(4)\ldots F(2(n-1))$ and $F(2n-1)!! = F(1)F(3)F(5)\ldots F(2n-1)$. The function $F(k)!!$ is set equal to unity if the argument k is less than or equal to zero. The above relations yields all the coefficients, $n = 1, 2, 3 \ldots$, in terms of $\langle 0|\alpha, F\rangle$ and $\langle 1|\alpha, F\rangle$. The coefficients $\langle 0|\alpha, F\rangle$ and $\langle 1|\alpha, F\rangle$ are fixed by normalizing the state $|\alpha, F\rangle$. If we choose $\langle 1|\alpha, F\rangle = 0$, the state $|\alpha, F\rangle$ involves the superposition of even number (Fock) states and represents the ENCS. If $\langle 0|\alpha, F\rangle = 0$, the state $|\alpha, F\rangle$, the superposition of odd number states, is the ONCS. We denote the ENCS as $|\alpha, F, +\rangle$, and the ONCS as $|\alpha, F, -\rangle$. In the linear limit, $F(\hat{n}) = 1$, the ENCS and ONCS become the ECS and the OCS, respectively. Depending on the form of $F(\hat{n})$ the ENCS and ONCS may exhibit many of the nonclassical features. It is interesting to note that the squeezed vacuum and the squeezed first excited state of the harmonic oscillator can be interpreted as the ENCS and ONCS, respectively. The squeezed vacuum is the ENCS when $F(\hat{n}) = 1/(1 + \hat{a}^\dagger\hat{a})$, and the squeezed first excited state is the ONCS with $F(\hat{n}) = 1/(2 + \hat{a}^\dagger\hat{a})$. In this paper we show that a class of ENCS and ONCS can be generated by the interaction of a harmonically trapped, two-level atom with two externtal laser fields of suitable frequency.

2. Description of the atom–field system

One of the interesting systems in quantum optics is the harmonically trapped and laser-driven ion wherein the interaction between the ion and the laser has nonlinear $\hat{n}$-dependence. This system has been studied in very many contexts: NCS [10], the vibronic Jaynes–Cummings interaction [21], the nonlinear Jaynes–Cummings interaction [22], the generation of even and odd coherent states [18], quantum signatures of chaos [23], quantum nondemolition measurements [24], quantum logic operations [25], engineering of Hamiltonians [26] and generation of ampltiude-squared squeezed states [27]. In this paper we show that a class of ENCS and ONCS can be generated in the centre-of-mass motion of a harmonically trapped ion via bichromatic laser excitation. We also study the nonclassical properties of the states produced.

We consider a single ion, having an electronic transition frequency ω and a lower (second) vibrational sideband with respect to that frequency, trapped in a harmonic potential of frequency ν. Two laser fields, tuned, repectively, to ω and the vibrational sideband transition frequency, interact with the ion. The Hamiltonian of this system in the optical rotating-wave approximation can be written as [18]

$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}}(t) \tag{8}$$

with

$$\hat{H}_0 = \hbar\nu\hat{a}^\dagger\hat{a} + \hbar\omega\hat{\sigma}_{22}. \tag{9}$$

The free-Hamiltonian $\hat{H}_0$ describes the free motion of the internal and external degrees of freedom and the interaction Hamiltonian $\hat{H}_{\text{int}}$,

$$\hat{H}_{\text{int}}(t) = \lambda[E_0\mathrm{e}^{[-\mathrm{i}(k_0\hat{x}-\omega t)]} + E_1\mathrm{e}^{[-\mathrm{i}(k_1\hat{x}-(\omega-2\nu)t)]}]\hat{\sigma}_{12} + \text{H.c.} \tag{10}$$

describes the interaction of the ion with the two laser fields. The operators $\hat{\sigma}_{ij}$ $(i, j = 1, 2)$ are the electronic flip operators corresponding to the transition $|j\rangle \to |i\rangle$, and $\hat{a}$ is the annihilation operator for the vibrational motion of the ion in the harmonic potential. The constant λ is the electronic coupling matrix element and k_0, k_1 are the wavevectors of the laser fields. The operator of the centre-of-mass position of the ion is

$$\hat{x} = \frac{\eta}{k_L}(\hat{a} + \hat{a}^\dagger) \tag{11}$$

where η is the Lamb–Dicke parameter and $k_L \simeq k_0 \simeq k_1$ is the wavevector of the driving laser field.

In the interaction picture, defined by the unitary transformation $\exp(-\frac{\mathrm{i}\hat{H}_0 t}{\hbar})$, the interaction Hamiltonian becomes

$$\hat{H}'_{\text{int}} = \exp\left(-\frac{\mathrm{i}\hat{H}_0 t}{\hbar}\right)\hat{H}_{\text{int}}\exp\left(\frac{\mathrm{i}\hat{H}_0 t}{\hbar}\right) \tag{12}$$

$$= \hbar\Omega_1\exp(-\eta^2/2)\hat{\sigma}_{12}\left[\sum_{k,l=0}^{\infty}\frac{(\mathrm{i}\eta)^{k+l}}{k!l!}\mathrm{e}^{\mathrm{i}(k-l-2)\nu t}\hat{a}^{\dagger k}\hat{a}^l + \frac{\Omega_0}{\Omega_1}\sum_{k,l=0}^{\infty}\frac{(\mathrm{i}\eta)^{k+l}}{k!l!}\mathrm{e}^{\mathrm{i}(k-l)\nu t}\hat{a}^{\dagger k}\hat{a}^l\right] + \text{H.c.} \tag{13}$$

where $\Omega_i = \frac{\lambda E_i}{\hbar}$ $(i = 1, 2)$ denotes the Rabi frequency of the two laser fields tuned to the electronic transition and the second sideband, respectively. In the rotating-wave approximation, neglecting terms rotating with frequencies ν or greater, the interaction picture Hamiltonian becomes

$$\hat{H}'_{\text{int}} = \hbar\Omega_1\exp(-\eta^2/2)\hat{\sigma}_{21}\hat{F} + \text{H.c.} \tag{14}$$

with

$$\hat{F} = \sum_{k=0}^{\infty}\frac{(\mathrm{i}\eta)^{2k+2}}{k!(k+2)!}\hat{a}^{\dagger k}\hat{a}^{k+2} + \frac{\Omega_0}{\Omega_1}\sum_{k=0}^{\infty}\frac{(\mathrm{i}\eta)^{2k}}{k!^2}\hat{a}^{\dagger k}\hat{a}^k. \tag{15}$$

3. Time evolution of the atom–field system

The time evolution of the system is governed by the master equation for the vibronic density operator $\hat{\rho}$,

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho} = -\frac{\mathrm{i}}{\hbar}[\hat{H}'_{\mathrm{int}}, \hat{\rho}] + \frac{\Gamma}{2}(2\hat{\sigma}_{12}\hat{\rho}'\hat{\sigma}_{21} - \hat{\sigma}_{22}\hat{\rho} - \hat{\rho}\hat{\sigma}_{22}) \tag{16}$$

where the second term is introduced to include the effect of spontaneous emission from the excited elctronic state with energy relaxation rate Γ, and

$$\hat{\rho}' = \tfrac{1}{2}\int_{-1}^{1} \mathrm{d}y\, W(y)\mathrm{e}^{\mathrm{i}\eta(\hat{a}+\hat{a}^{\dagger})y}\hat{\rho}\mathrm{e}^{-\mathrm{i}\eta(\hat{a}+\hat{a}^{\dagger})y} \tag{17}$$

accounts for changes in the vibrational energy due to spontaneous emission. $W(y)$ gives the angular distribution of spontaneous emission. The steady state solution $\hat{\rho}_s$ of equation (16) is obtained by setting $\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho} = 0$. The steady state attained depends on the initial state of the system. It is important to note that the master equation includes the effect of electronic damping while the vibronic damping is taken to be negligible. This, in turn, would imply that the electronic part of the steady state solution contains only the ground state. To solve for $\hat{\rho}_s$, we make the ansatz that $\hat{\rho}_s$ is given by

$$\hat{\rho}_s = |1)|\zeta\rangle\langle\zeta|(1| \tag{18}$$

where $|1)$ is the electronic ground state and $|\zeta\rangle$ is the vibrational state of the ion, then the state $|\zeta\rangle$ obeys

$$\hat{F}|\zeta\rangle = 0. \tag{19}$$

Using $\hat{F}$ given by (15) we get

$$\langle n+2|\zeta\rangle = \frac{\Omega_0}{\Omega_1\eta^2}\frac{(n+1)(n+2)L_n^0(\eta^2)}{\sqrt{(n+1)(n+2)}L_n^2(\eta^2)}\langle n \mid \zeta\rangle \tag{20}$$

where L_n^m is an associated Laguerre polynomial defined by

$$L_n^m(x) = \sum_{l=0}^{n}\begin{pmatrix} n+m \\ n-l \end{pmatrix}\frac{(-x)^l}{l!}. \tag{21}$$

The numbers $\langle n+2|\zeta\rangle$ are the expansion coefficients for the state $|\zeta\rangle$ in the Fock states basis. Comparing with (5) indicates that the state $|\zeta\rangle$ is an ENCS or ONCS with

$$\alpha = \frac{\Omega_0}{\Omega_1\eta^2} \tag{22}$$

and

$$F(n) = L_n^2(\eta^2)[(n+1)(n+2)L_n^0(\eta^2)]^{-1}. \tag{23}$$

In the limit $\eta \to 0$ the function $F(n)$ becomes $\frac{1}{2}$ for all n. Hence in the small-η limit the ENCS and ONCS become the ECS and OCS, respectively.

4. Properties of the ENCS and ONCS

If the initial state of the ion is a combination of even (odd) number states then the state of the system at later times will only involve a superposition of even (odd) number states as the

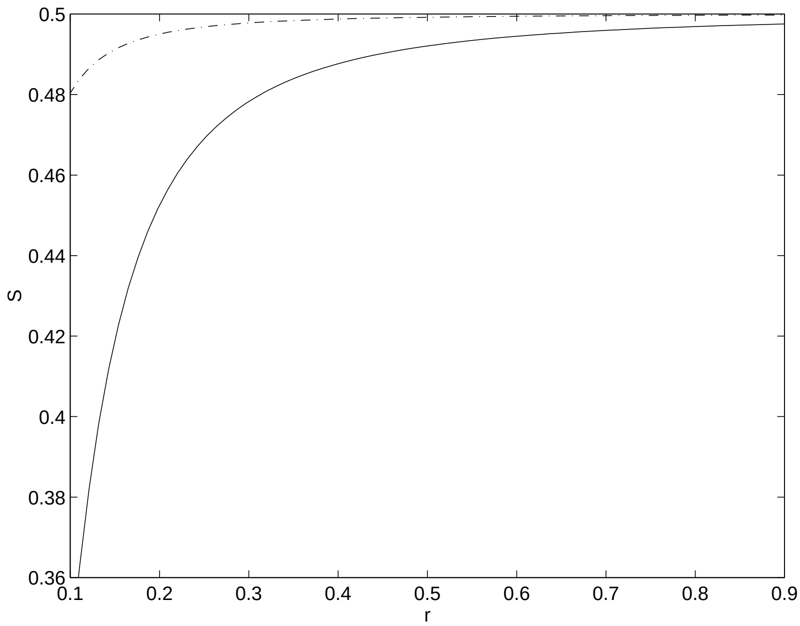

Figure 1. Uncertainty S, $\langle p^2\rangle - \langle p\rangle^2$, in p as a function of r for $\frac{\Omega_0}{\Omega_1} = 0.001$ (solid curve) and $\frac{\Omega_0}{\Omega_1} = 0.0001$ (dashed curve) for the state $|\alpha, F, +\rangle$. r represents real η.

master equation (16) contains only even powers of $\hat{a}$ and $\hat{a}^\dagger$. If the initial state of the ion is the vacuum state then the stationary state of the system is given by

$$|\alpha, F, +\rangle = N \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{(2n)!}F(2n-2)!!}|2n\rangle \qquad N^{-1} = \sqrt{\sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{(2n)!(F(2n-2)!!)^2}} \tag{24}$$

where α and $F(n)$ are defined by equations (22) and (23), respectively. This state is the ENCS for the vibrational motion of the centre-of-mass of the ion in the harmonic potential. The behaviour of the expansion coefficients $\langle n|\alpha, F, +\rangle$ is highly oscillatory becoming zero for odd n. This oscillatory behaviour is one of the nonclassical features.

The ECS exhibits squeezing in the p-quadrature which is defined as $\mathrm{i}(\hat{a}^\dagger - \hat{a})/\sqrt{2}$. For the ENCS the expectation values of $\hat{a}$ and $\hat{a}^\dagger$ become zero and the uncertainty in p is given by

$$\langle(\triangle\hat{p})^2\rangle = \langle\hat{p}^2\rangle - \langle\hat{p}\rangle^2 \tag{25}$$

$$= \tfrac{1}{2}[1 + 2\langle\hat{a}^\dagger\hat{a}\rangle - 2\langle\hat{a}^2\rangle] \tag{26}$$

where the expansion coefficients of the ENCS in the harmonic oscillator basis are taken to be real. In figure 1 we have shown the uncertainty in p as a function of η for the states defined by (24). From figure 1 it is clear that the uncertainty in p is less than that of the vacuum state value of 0.5 indicating that the states exhibit squeezing. As η increases the uncertainty in p approaches that of the vacuum state. The reason for this behaviour is the following. As η increases the occupation number distribution $p(n) = |\langle n|\alpha, F, +\rangle|^2$ starts peaking near $n = 0$. To make this explicit we have shown in figure 2 the occupation number distribution $p(n)$ as a function of n for various values of η.

The occupation number distribution for the ECS is always super-Poissonian, that is variance in $\hat{n}$ is larger than its mean. A state is said to exhibit sub-Poissonian statistics if

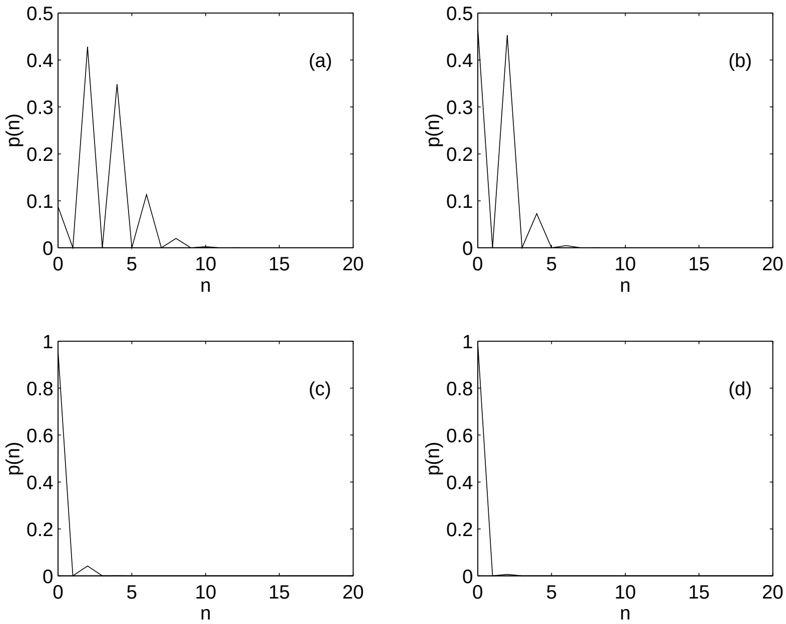

Figure 2. Occupation number distribution $p(n)$ as a function of n for the state $|\alpha, F, +\rangle$ for various η values and $\frac{\Omega_0}{\Omega_1} = 0.0001$. (*a*) $\eta = 0.008$, (*b*) $\eta = 0.012$, (*c*) $\eta = 0.02$, and (*d*) $\eta = 0.1$.

the q parameter [28], defined as

$$q = \frac{\langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2}{\langle \hat{n} \rangle} - 1 \tag{27}$$

is negative. Negative q indicates that the state is nonclassical. For the coherent states q is zero. For the ENCS of (24) the ditribution $p(n)$ can have negative q for suitable values of η and α. In figure 3 we have shown the variation of q with respect to the Lamb–Dicke parameter η for two different values of α. It is evident that the ENCS can have features that are absent in the ECS.

If the initial state of the ion is the first excited state of the harmonic trap then the state of the system at later times will involve only odd number states. The resultant stationary state of the system is an ONCS given by

$$\begin{aligned} |\alpha, F, -\rangle &= N \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{(2n+1)!}F(2n-1)!!} |2n+1\rangle \\ N^{-1} &= \sqrt{\sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{(2n+1)!(F(2n-1)!!)^2}} \end{aligned} \tag{28}$$

where $F(n)$ and α are again defined by equations (23) and (22), respectively. As in the case of ENCS the behaviour of the occupation number distribution of the ONCS, equation (28), is oscillatory becoming zero for even n. The occupation number distribution $p(n)$ of the ONCS is sub-Poissonian. Figure 4 shows the q parameter as a function of α for the ONCS of (28). It is clear that the states are sub-Poissonian. It is interesting to note that the q value for large values η approaches the value of that of the first excited state of the harmonic oscillator. The reason being that the occupation number distribution becomes concentrated at $n = 1$ as η increases.

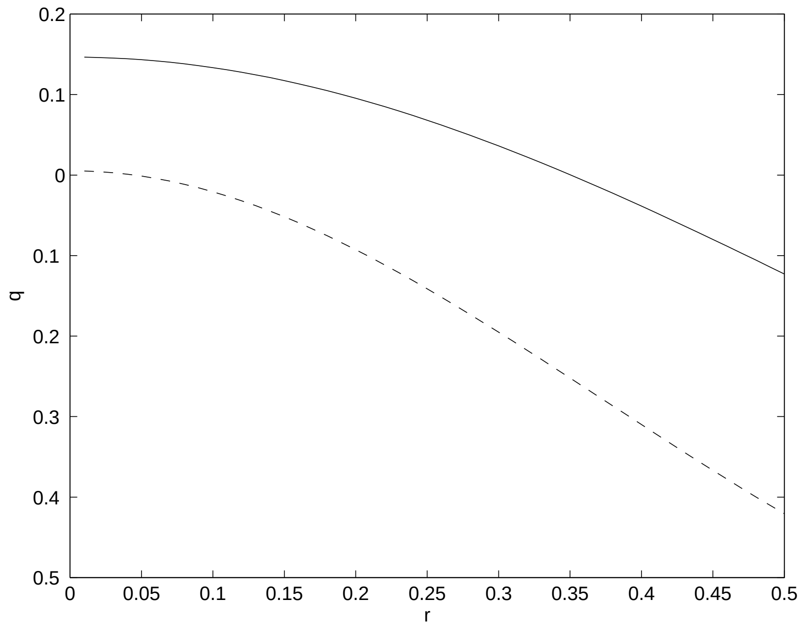

Figure 3. Mandel's q parameter as a function of r. The solid curve corresponds to $\alpha = 1.0$ and the dashed curve to $\alpha = 2.0$.

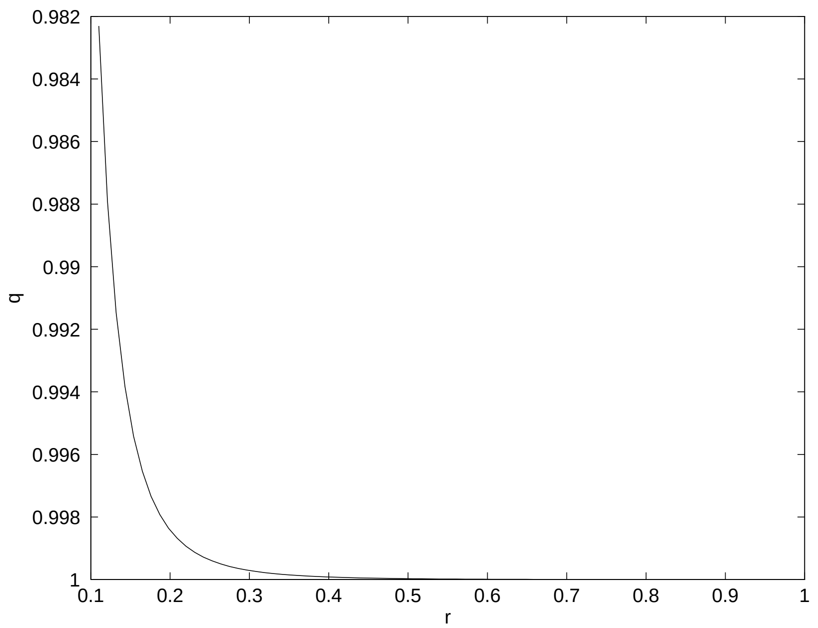

Figure 4. Mandel's q parameter as a function of r for $\frac{\Omega_0}{\Omega_1} = 0.001$ for the state $|\alpha, F, -\rangle$. r represents real η.

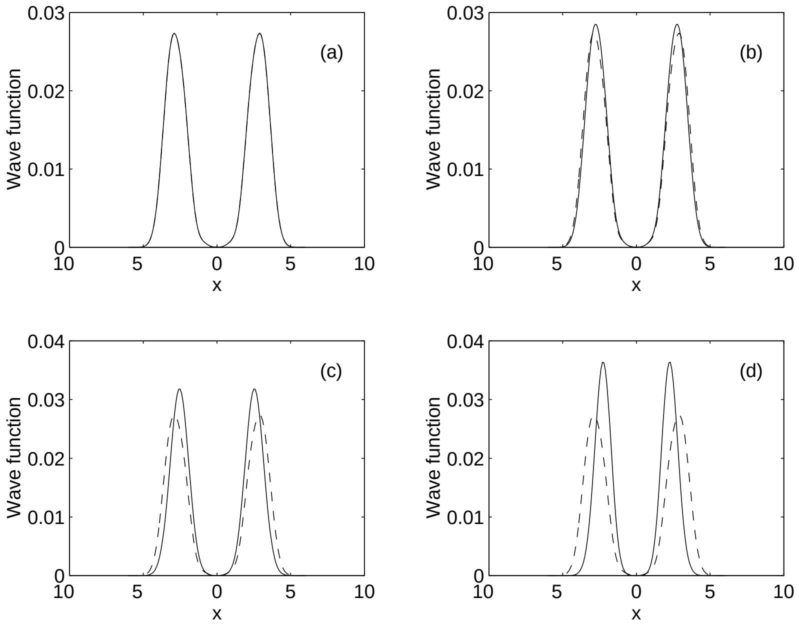

Figure 5. Position basis wavefunction corresponding to the state $|\alpha, F, +\rangle$ of (24) with $\alpha = 2.0$. (*a*) $\eta = 0.0$, (*b*) $\eta = 0.15$, (*c*) $\eta = 0.30$ and (*d*) $\eta = 0.50$. The dashed curves correspond to the ECS and are the same as case (*a*).

It is interesting to see how good the ENCS is in approximating the Schrodinger cat states. We have plotted the wavefunction of the ENCS in figure 5. It is seen that the ENCS is indeed close to the ECS, which is a cat state. The aprroximation becomes poorer as the value of η increases. A similar trend is found in the case of the ONCS too. They approximate the OCS for small values of η. So far we have discussed the steady state resulting from the evolution of an initially pure state. Since the vibronic damping is neglected, the problem of decoherence does not arise. Nevertheless, it is possible to assume that the system evolves from an initially mixed state for the vibronic part. In such a case the steady state solution is

$$\hat{\rho}_s = W_e|1)|\alpha, F, +\rangle\langle\alpha, F, +|(1| + W_o|1)|\alpha, F, -\rangle\langle\alpha, F, -|(1|. \qquad (29)$$

Here W_e and W_o are weights of the even and odd number states in the initial state.

5. Summary

In conclusion, we have shown that a class of ENCS and ONCS can be generated in the centre-of-mass motion of a trapped and bichromatically laser driven ion. These ENCS and ONCS are nonclassical. The ENCS exhibits squeezing while the ONCS exhibits sub-Poissonian statistics. Both the ENCS and ONCS have an oscillatory occupation number distribution. The ENCS exhibits sub-Poissonian statistics for suitable values of the parameters η and α while the ECS is always super-Poissonian. For small values of η the ENCS and ONCS are very close to the ECS and OCS, respectively. It is the occurence of the two physical parameters η and α that helps in manoeuvring the coefficients so that the resultant states have properties very different from those of the ECS and the OCS.

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